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SPECIAL NUCLEAR MATERIAL INVENTORY SAMPLING PLANS

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BACKGROUND

Since their introduction in 1942, item sampling procedures have been standard quality assurance practice. Instead of examining an entire inventory, only some of the items are examined and the results are extrapolated to the entire inventory. In the nuclear industry, single sampling plans are commonly used with facilities relying on tables published in WASH-1210 (Ref. 1) for determining sample size.

DOE supports such sampling of special nuclear material (SNM) inventories. DOE Order 5630.7 states:

> "Operations Offices may develop and use statistically valid sampling plans appropriate for their site-specific needs."

The benefits of item sampling for nuclear facilities operations include reduced worker exposure to radiation and reduced work load.

This paper presents improved procedures for obtaining "statistically valid sampling plans" that maximize these benefits. Specifically, we describe the double sampling concept and methods for developing optimal double sampling plans. Comparisons between the sample sizes for double sampling plans and single sampling plans show the double plans to be worth the added complexity.

A double sampling plan is regarded as superior to a single sampling plan because frequently the average number of items sampled (ASN) under the double plan is 10-50% less than under a single plan. (For optimal double sampling plans, ASN is never greater the the single plan sample size.) Double sampling plans can save the nuclear industry a comparable amount of inspection time and employee radiation exposure.

Tables for determining double sampling plans can be found in Mil. Std. 105D, which was developed originally in 1942, with its most recent version available in 1963 (Ref. 2). Several computer programs for this purpose have been published as well.[3,4] However, neither the original tables, nor the available computer programs are totally satisfactory.

We describe a new algorithm that is satisfactory for finding optimal double sampling plans and choosing appropriate detection and false alarm probabilities. The algorithm also extends sampling plan generation techniques to account for *a priori* knowledge of the expected number of inventory defects. Moreover, its running times are suitable for use on a personal computer.

SAMPLING PLAN CONCEPTS

A sampling plan attempts to satisfy two desired levels of protection. A probability of at least $1 - \alpha$ of accepting N items (the size of the inventory) is desired if the proportion of defective items in N is at the acceptable quality level (AQL); and a probability of acceptance of no more than β is desired if the proportion defective is at the rejectable quality level (RQL). The AQL is chosen to be the highest percent defectives that an inventory holder would find acceptable as an average. The RQL is the lowest percent defectives acceptable as an average. If a known typical quality level (TQL) is less than the AQL or more than the RQL, considerable savings in sample size can be made for double plans.

DOE Order 5630.7 calls for $\beta \leq 0.20$ for attribute sampling tests and 0.50 for variables tests, but the DOE area Operations Offices may seek substantially smaller values. Clearly we, as inventory holders, want α to be very small to minimize false rejections; but, in practice, 0.05 is often used for safeguards applications.

Single Sampling Plan Defined

With single sampling attribute plans, a sample of size n is chosen randomly and examined for defective items. If the number of defects found is greater than or equal to some limit C, the inventory is rejected; otherwise, it is accepted. Line 1 of Table I shows a single sampling plan for N = 2000. With an AQL of 2% and RQL of 7% at target probabilities of 0.05 and 0.20 for α and β, respectively, the single sampling plan that comes closest to the targets is:

(1) Sample 94 items; then

(1a) Reject the inventory if 5 or more defects are discovered.

The α (inventory holder's) risk is 0.037 and the β risk is 0.198.

Double Sampling Plan Defined

Without changing AQL, RQL, or the target values for α and β, the average sample size can be substantially reduced by using a double sampling plan as demonstrated by Table I, line 2. The procedure for the corresponding double sampling plan is illustrated as follows:

(1) Sample 47 items.

(1a) If the number of defects is 1 or less, accept the inventory.

(1b) If the number of defects is 4 or more, reject the inventory.

(2) If the number of defects is 2 or 3, sample an additional 61 items (totalling 108).

(2a) If the total number of defects is 4 or less, accept the inventory.

(2b) If there are more than 4 defects, reject the inventory.

Thus, a double sampling plan is designated by:

3 acceptance numbers c_1, c_2, and c_3 ($c_1 \leq c_2 \leq c_3$), and

2 sample sizes n_1 and n_2.

A sample of size n_1 is chosen from a given lot of N items.

Accept the lot if there are c_1 or fewer defective items in n_1. Reject the lot if it contains c_2 or more defective items.

Sample an additional n_2 items if the number of defects in n_1 is greater than c_1 and less than c_2.

Accept the lot if in the n_1 plus n_2 sampled items, there are c_3 or fewer defects. Reject if there are greater than c_3 defects.

Under the double sampling plan, the average number of samples actually inspected (ASN) is only 60.9, or 35% fewer than the single sampling plan. If we know a priori that the average inventory has only 1% defective items (the usual assumption is that the inventory contains an average of AQL defects--2% in this example), there is a better double sampling plan with an ASN of only 48.6. Thus, knowing TQL = 1.0% can reduce the expected number of samples 48% below the single plan; however, the α risk increases from 0.037 to 0.048, nearly the 0.05 target.

FINDING DOUBLE SAMPLING PLANS

The literature contains tables and algorithms for computing double sampling plans.[2-4] Those methods contain simplifications that are not optimal. For example, Mil. Std. 105D requires $n_1 = n_2$ and recently published computer programs either require $n_1 = k*n_2$ (k is an integer),[3] or uncouple n_1 and n_2 but require $c_2 = c_3$ (Ref. 4). Thus, the feasible solution space for the tuple (n_1, n_2, c_1, c_2, c_3), which we

TABLE I. THE EFFECTIVENESS OF DOUBLE SAMPLING PLANS*

Plan	Inventory	α Risk	β Risk	Sample 1	Sample 2	Accept $\leq c_1$	Reject $\geq c_2$	Accept $\leq c_3$	TQL	ASN
Single	2000	0.037	0.198	94	-	4	5	-	-	94
Double	2000	0.050	0.198	47	61	1	4	4	0.02	60.9
Double	2000	0.048	0.200	29	77	0	4	4	0.01	48.6

*α limit = 0.050, β limit = 0.20, AQL = 0.02, and RQL = 0.07.

simply call a double sampling plan (DSP), is restricted before searching for the best plan.

In many cases the efficiency is nearly the same when choosing $c_2 = c_3$ (see Ref. 5). However, we will demonstrate that this can be an unsuccessful strategy. The optimally efficient DSP for sampling environments we have examined typically falls outside the region defined by $n_1 = k*n_2$ and often violates $c_2 = c_3$.

We developed an algorithm for finding plans that are optimal with respect to ASN and implemented the algorithm as VAX and PC-based software. Our algorithms produce even better results for the nuclear industry because our search implements TQL for computing the ASN. (The DOE-suggested AQL of 1.0% for the Los Alamos plutonium facility, for example, is well above the TQL experienced there. A smaller TQL results in an optimal double sampling plan with lower ASN.)

DOUBLE SAMPLING PLAN SEARCH ALGORITHM

Our algorithm first bounds the solution space for double sampling by using the optimal single sampling plan, which it also computes, as an upper bound on ASN, n_1, $n_1 + n_2$, and c_1. As the search progresses, the solution space is repeatedly narrowed by identifying infeasible and dominated regions. A selection of locally optimal sampling plans are printed, and finally the globally optimal plan is identified.

Nomenclature and Computations

To explain the algorithm, let us first define and reiterate some nomenclature. We use the following parameters as inputs to the search:

N - The number of inventory items.
TQL - "Typical Quality Level", expected or "normal" defect fraction.
AQL - "Acceptable Quality Level", # defects/lot size.
α - Ceiling on probability of rejecting a lot of AQL quality.
RQL - "Rejection Quality Level", # defects/lot size.
β - Ceiling on the probability of accepting a lot of RQL quality.

The search produces a series of outputs of the form {ASN, n_1, n_2, c_1, c_2, c_3, P_a and P_b} where:

ASN - The average or expected number of sample taken. Because the second sample is not always needed, $n_1 < ASN \leq n_1 + n_2$.
n_1 - The size of the first sample.
n_2 - The size of the second sample, if taken.
c_1, c_2, c_3 - The number of defects permitted for acceptance/rejection limits.
P_a - The actual probability of rejecting a lot of AQL.
P_b - The actual probability of accepting a lot of RQL.

The acceptance/rejection criteria are defined more formally in terms of the number of defects d_1 in n_1 and the number of defects d_2 in n_2.

Acceptance Criteria:

$(d_1 \leq c_1)$ or

$(d_1 < c_2$ and $d_1 + d_2 \leq c_3)$.

Rejection Criteria:

$(d_1 \geq c_2)$ or

$(d_1 > c_1$ and $d_1 + d_2 > c_3)$.

Objective:

Find a tuple {n_1, n_2, c_1, c_2, c_3} to minimize the ASN for sampling an inventory of size N with an expected defect fraction TQL.

Subject to:

$P_a \leq \alpha$ with an inventory of actual quality AQL.
$P_b \leq \beta$ with an inventory of actual quality RQL.

For any DSP tuple, we can compute P_a, P_b, and ASN directly from the Hypergeometric distribution (see Ref. 6, p. 210). Goldman shows the basic formulation for double sample plan calculations (Ref. 7, p. 10):

$$P(\text{acceptance given D defects in N}) = \frac{\sum_{d=0}^{c_1} \binom{D}{d}\binom{N-D}{n_1-d}}{\binom{N}{n_1}}$$

$$+ \sum_{d=c_1+1}^{c_2-1} \sum_{j=0}^{c_3-d} \frac{\binom{D}{d}\binom{N-D}{n_1-d}}{\binom{N}{n_1}} \frac{\binom{D-d}{j}\binom{N-n_1-D+d}{n_2-j}}{\binom{N-n_1}{n_2}} .$$

In terms of this hypergeometric function, which we designate as $H(N,D,n_1,n_2,c_1,c_2,c_3)$, P_a and P_b are

$$P_a = P(\text{rejection given AQL} \times \text{N defects in N})$$
$$= 1 - P(\text{acceptance given AQL} \times \text{N defects in N})$$
$$= 1 - H(N, AQL \times N, n_1, n_2, c_1, c_2, c_3)$$

$$P_b = P(\text{acceptance given RQL} \times \text{N defects in N})$$
$$= H(N, RQL \times N, n_1, n_2, c_1, c_2, c_3)$$

ASN is computed from the same function as:

$$
\begin{aligned}
ASN = {} & n_1 \\
& + n_2 \times P[\text{no decision on sample 1 given TQL} \times N \text{ defects in } N] \\
= {} & n_1 + n_2 \times (1 - P[\text{acceptance on first sample}] \\
& - P[\text{rejection on first sample}]) \\
= {} & n_1 + n_2 \times [1 - H(N, TQL \times N, n_1, 0, c_1, c_1+1, c_1+1) \\
& - (1 - H(N, TQL \times N, n_1, 0, c_2, c_2+1, c_2+1)] \\
= {} & n_1 + n_2 \times [\, H(N, TQL \times N, n_1, 0, c_2, c_2+1, c_2+1) \\
& - H(N, TQL \times N, n_1, 0, c_1, c_1+1, c_1+1) \,]
\end{aligned}
$$

Our computer programs take advantage of the similarity of all three expressions, reusing results whenever practical. In fact, during a sequential search in which one parameter is varied, all the expressions can be computed recursively.

In addition to the reuse concept, the key to a successful search strategy is to recognize infeasible regions and dominated regions and eliminate them from the search. The first such dominated region is produced by the optimal single sampling plan.

Single Sampling Plan Solution

We find the optimal single sampling plan $\{n, C\}$ for AQL, RQL, N, α, and β as follows.

Require $0.0 < AQL < RQL < 1.0$.

(a) Let C, the maximum number of defects in an accepted sample, initially be 0.

(b) Establish upper (N_u) and lower (N_l) bounds on n, the sample size. Initially, $N_l = C$ and $N_u = N - (RQL \times N) + C + 1$.

(To understand the upper bound, observe that with the sample size at N_u, an inventory with $N \times RQL$ defects will be accepted with zero probability ($P_b = 0$), because we will always find at least $C + 1$ defects and reject the inventory. If $P_a \leq \alpha$ with this sample size, the solution is feasible. Decreasing n will reduce P_a and improve the solution, provided that P_b, which will increase, remains $\leq \beta$. The lower bound is understood through similar reasoning.)

(c) Set $n = (N_l + N_u + 1)/2$ midway between the two limits, and compute P_a and P_b.

(d) If the plan is both α-infeasible (that is, $P_a > \alpha$) and β-infeasible, there is no feasible solution for this C. Continue with step (j) below.

(e) If the plan is just β-infeasible, increase N_l to n.

(f) Otherwise, decrease N_u to n.

(g) Again, set $n = (N_l + N_u + 1)/2$ midway between the two limits, and compute P_a and P_b.

(h) If n is not at either of the limits yet, repeat step (d) above.

(i) If the plan is both α and β-feasible, this is the optimal single sampling plan. Stop.

(Note: there may be feasible single sampling plans for larger values of C, but they will have larger sample sizes as well. Our search decreases n to the minimum that was still β-feasible. For a fixed n, increasing C will increase P_b further.)

(j) Increase C by 1. If $C \leq AQL \times N$, repeat step (b) above.

(Note that for C at this upper limit, $P_a = 0$ because an inventory of AQL quality does not contain enough defects to be rejected even if the sample contains them all. Increasing C further provides no added benefits--the plan is already α-feasible. If the optimal single plan has not been found, then the current plan must be β-infeasible. Increasing C will only increase P_b further.)

(k) Otherwise there is no feasible single sampling plan.

Double Sampling Plan Solution

The current values of n and C from the single sampling plan search provide initial values for a double sampling plan search. Specifically, n is an upper bound on the ASN (and n_1) of a successful double plan, and C bounds c_1 via $c_1 + 1 \leq C$. By our definition of terms, we also have $c_2 > c_1 + 1$ and $c_2 \leq c_3$. (If $c_2 = c_1 + 1$, we have a single sampling plan.)

The double sampling plan search then proceeds as follows:

(a) Set lower and upper limits on c_1 as $C_{1l} = 0$, $C_{1u} = C - 1$.

(b) Start with an initial $c_1 = C_{1l}$.

(c) Set lower and upper limits on c_2 as $C_{2l} = C_l + 2$, $C_{2u} = C_{1u} + 2$.

(d) Start with an initial $c_2 = C_{2l}$.

(e) Set lower and upper bounds on c_3 as $C_{3l} = \text{maximum}(c_2, C)$, $C_{3u} = AQL \times N + 1$. (See note for (j) above.)

(f) Start with an initial $c_3 = C_{3l}$.

(g) Set lower and upper limits on n_1 as $N_{1l} = C_2$, $N_{1u} = n - 1$.

(h) Initially, let $n_1 = N_{1l}$. Maintain a state indicator, initially set to state -1.

(i) Find the minimum n_2 that satisfies the Acceptance/Rejection criteria for the current n_1, c_1, c_2, c_3. This is done via a binary search process analogous to that described under the single sample plan search. This search returns with values for n_2, p_2 = P[second sample is required], P_a, P_D, and a new state indicator code as follows:

State 0 - a feasible solution was found.
State 1 - the entire n_2 range is α-infeasible.
State 2 - the entire n_2 range is β-infeasible.
State 3 - the entire n_2 range is both α- and β-infeasible.
State -1 - part of the n_2 range is both α- and β-infeasible. The rest is either α- or β-infeasible.

(Note that n_2 is lower bounded by N_{2l} = maximum($C_3 - C_1$, 1) and upper bounded by $N_{2u} = N - N_1 - N \times RQL + C_1$.)

(j) If the current DSP tuple is State 1, increase c_3 and continue with step (n).

(k) If the tuple is in State 3, increase c_2 if it enables a decrease in c_3 (i.e., $c_2 + 1 < c_3$). If c_3 cannot be decreased, decrease c_1 if it enables a decrease in c_2 (i.e., $c_1 + 2 < c_2$). If c_2 cannot be increased, the search is complete; stop. Otherwise, continue with step (n).

(l) If the tuple is in any other state except 0, set n_1 to the midpoint of N_{1l} and N_{2l}. Begin a binary search on n_1, adjusting the upper limit whenever the n_2 search returns State 1 and the lower limit when it returns to State 2. If State 0 is encountered, continue with step (m). Increase c_3 and continue with step (n) if State 3 occurs. Otherwise, if $n_1 > n_2$, set N_{1u} to n_1; if $n_1 < n_2$, set N_{1l} to n_1; if $n_1 = n_2$, increase c_3 and continue with step (n).

(m) If the tuple is in State 0, it is feasible. Perform a single step search on n_1 toward each limit for n_1, using the state code returned by the n_2 search routine at each step to determine when to terminate the search in a particular direction. (Note that there may be a second region of feasibility that must be searched. By examining the slope of p_2 with respect to n_1, the direction of the second region, if it exists, can be determined. A repeat of the binary search of step (l) finds the region.) Retain the best double sampling plan found during this phase of the search. When complete, compare the retained plan with best double sampling plan found so far (i.e, with other c_1, c_2, c_3 parameters). Save the plan with the lower ASN. Increase c_3 and continue with step (n).

(n) If $c_3 \le C_{3u}$, repeat step (f). Otherwise, increase c_2.

(o) If $c_2 \le C_{2u}$, repeat step (e). Otherwise, increase c_1.

(p) If $c_1 \le C_{1u}$, repeat step (c). Otherwise, stop.

SELECTED RESULTS

Tables II and III show the ASN for several single and double sampling plans found by this algorithm. Note, for example, that the ASN for the best double sampling plan in Table III is 28% better than the single plan and about 6% better than the first plan listed by Olorunniwo and Salas. However, a facility with a TQL less than the AQL of 2% might favor a plan with a smaller first sample and larger second sample. Table IV shows several plans for a TQL of 1.0%, inventory of 1000, AQL of 2.5%, and RQL of 5.0%. The best plan now has an ASN 67% below the single sampling plan. Moreover, note that $c_2 \ne c_3$ for this plan.

TABLE II. SAMPLING PLANS* LISTED BY OLORUNNIWO AND SALAS
(Ref. 4, Listing ²)

Plan	Inventory	α Risk	β Risk	Sample 1	Sample 2	Accept ≤ c_1	Reject ≥ c_2	Accept ≤ c_3	TQL	ASN
Double	500	4.84	10.64	38	76	0	4	4	0.02	79.5
Double	500	5.36	9.48	39	78	0	4	4	0.02	82.3
Double	500	2.04	9.91	43	86	0	5	5	0.02	94.2
Double	500	4.48	9.33	54	108	1	5	5	0.02	85.6
Double	500	3.71	9.47	72	144	2	6	6	0.02	95.3
Double	500	0.08	9.97	89	178	3	9	9	0.02	103.8

*α limit = 0.0536, β limit = 0.1064, AQL = 0.02, and RQL = 0.07.

TABLE III. SAMPLING PLANS* FOUND BY THE AUTHORS' SEARCH ALGORITHM

Plan	Inventory	α Risk	β Risk	Sample 1	Sample 2	Accept ≤ c_1	Reject ≥ c_2	Accept ≤ c_3	TQL	ASN
Single	500	3.83	10.55	105	0	4	5	0	0.02	105.0
Double	500	3.38	10.61	35	117	0	3	6	0.02	92.6
Double	500	5.32	10.47	36	82	0	4	4	0.02	79.1
Double	500	2.81	10.52	34	107	0	5	5	0.02	88.4
Double**	500	5.27	10.55	58	56	1	4	4	0.02	75.1
Double	500	2.87	10.60	54	90	1	5	5	0.02	80.4
Double	500	5.22	10.63	72	68	2	4	5	0.02	80.2
Double	500	2.96	10.58	72	74	2	5	5	0.02	83.5
Double	500	3.15	10.62	89	56	3	5	5	0.02	92.6

*α limit = 0.0536, β limit = 0.1064, AQL = 0.02, and RQL = 0.07.

**Optimal double sampling plan.

TABLE IV. SELECTED DOUBLE SAMPLING PLANS*

Plan	Inventory	α Risk	β Risk	Sample 1	Sample 2	Accept ≤ C1	Reject ≥ C2	Accept ≤ C3	TQL	ASN
Single	1000	0.048	0.049	379	–	13	14	–	0.01	379
Double	1000	0.050	0.050	97	376	1	6	16	0.01	191.7
Double	1000	0.049	0.050	96	361	1	7	15	0.01	185.6
Double**	1000	0.048	0.050	124	338	2	9	15	0.01	163.3
Double	1000	0.050	0.049	192	189	3	12	13	0.01	212.1

*α limit = 0.050, β limit = 0.050, AQL = 0.025, and RQL = 0.05.

**Optimal double sampling plan.

We believe that double sampling plans can reduce labor and radiation exposure to such a great extent that it behooves facilities to switch to these procedures. Although our search program is relatively complex, the software is easy to use, its performance on a personal computer is quite fast, and thus far it is a reliable, useful tool. These programs will be made available to requestors in early 1988, and we invite inquiries at the present time.

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